Type inference in the multirate audio DSP language Faust

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SYNCHRON 2012

FAUST : Functional AUdio STream

Language developed at GRAME (Lyon) since 2003.

ANR Astrée 2009-2011 (MINES ParisTech, IRCAM and U. St Etienne).

Compiled language, real-time audio applications.

Audio synthesis, treatments; interactive applications.

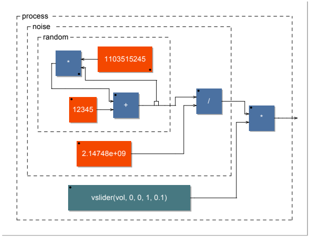
Work at sample-level (typically 44.1 kHz).

Plan

Overview

Domain-specific language, audio digital signal processing.

- synchronous
- purely functionnal
- textual block-diagram description
- statically typed




```
random  = +(12345)~*(1103515245);
noise   = random/2147483647.0;

process = noise * vslider("vol",0,0,1,0.1);
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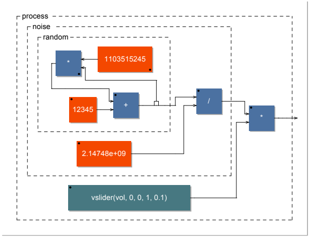


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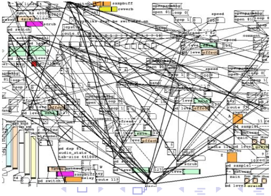
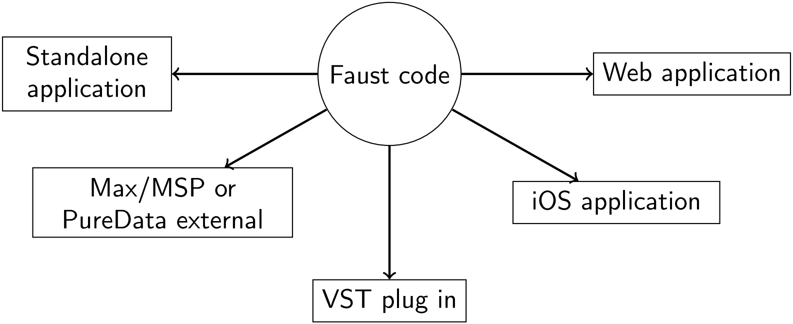
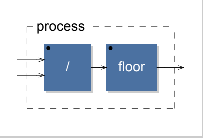


- Automatic generation of optimized C++ code
- Online compiler: `http://faust.grame.fr/`
- Multi-target compilation



CoreFaust

Block diagrams are built using 5 composition operators:




```
/ : floor
```

$$y(t) = \left\lfloor \frac{x_1(t)}{x_2(t)} \right\rfloor$$

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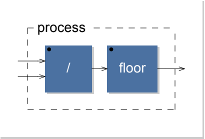

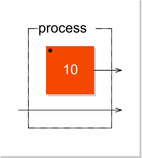

`/ : floor`

$$y(t) = \left\lfloor \frac{x_1(t)}{x_2(t)} \right\rfloor$$

`10 , _`

$$y(t) = (10, x(t))$$

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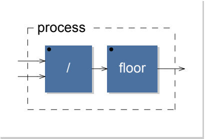
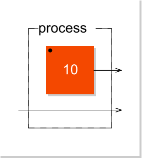
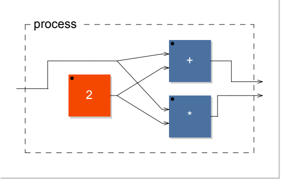






`/ : floor`

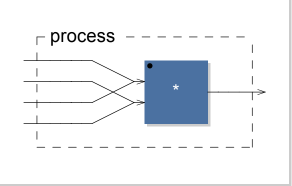
`10 , _`

`_,2 <: +,*`

$$y(t) = \left\lfloor \frac{x_1(t)}{x_2(t)} \right\rfloor$$

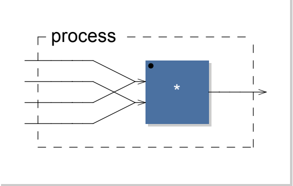
$$y(t) = (10, x(t))$$

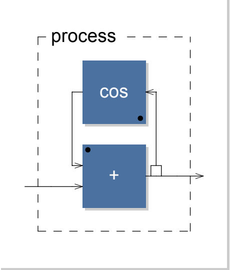
$$y(t) = (x(t) + 2, 2x(t))$$




```
_,_,_,_ :> *
```

$$y(t) = (x_1(t) + x_3(t)) * (x_2(t) + x_4(t))$$





`_,_,_,_ :> *`

`+ ~ cos`

$$y(t) = (x_1(t) + x_3(t)) * (x_2(t) + x_4(t))$$

$$\begin{cases} y(0) = x(0) + cos(0) \\ y(t) = x(t) + cos(y(t-1)) \end{cases}$$

Faust language

High-level functional language with lambdas, libraries, pattern-matching, infix notations, local environments...

```
fact(n) = case {
            (0) => 1;
            (n) => (n,fact(n-1)) : *;
};

q(x,y) = floor(x/y);   //stands for x,y : / : floor

mix = \(n).(par(i,n,_) :> _);
```

Expressiveness

Faust is Turing-complete.

The current version is monorate; wires carry only scalar signals.

We need

- different rates
- more complex data structures (vectors, matrices...)

to deal with multirate signal processing or spectral analyses.

Types

$$\text{BasicType} = \{\text{Int}, \text{Float}\}$$
$$\text{Interval} = \mathbb{R}^\omega \times \mathbb{R}^\omega \quad (\mathbb{R}^\omega = \mathbb{R} \cup \{-\omega, \omega\})$$
$$\text{Frequency} = \mathbb{Q}_+$$

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$$\begin{array}{rcll} \tau \in \text{Type} & = & \text{BasicType} \times \text{Interval} & \textit{e.g.}\ \text{Float}[0,1] \\ & \mid & \mathbb{N}^* \times \text{Type} & \textit{e.g.}\ \textit{vector}_n(\tau) \end{array}$$

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Subtyping rules :

$$[x,y] \subset [x',y'] \Rightarrow b[x,y]^f \subset b[x',y']^f$$

$$\textit{Int}[x,y]^f \subset \textit{Float}[x,y]^f$$

$$\tau^0 \subset \tau^f$$

$$\tau \subset \tau' \Rightarrow \textit{vector}_n(\tau)^f \subset \textit{vector}_n(\tau')^f$$

Initial environment

Associates to predefined identifiers their input and output types.

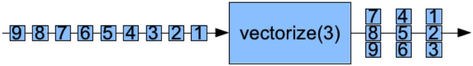
$$
\begin{aligned}
T(\_) &= \Lambda f : Rate.\tau : Type.(\tau^f) \rightarrow (\tau^f) \\
T(0) &= \Lambda f : Rate.() \rightarrow (Int[0,0]^0) \\
T(+) &= \Lambda f : Rate.\tau : Type.\tau' : Type.(\tau^f, \tau'^f) \rightarrow (\tau + \tau')^f
\end{aligned}
$$

Binary operations are well formed if :

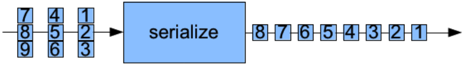
$$\exists \overline{\tau} \,/\, (\tau \subset \overline{\tau} \wedge \tau' \subset \overline{\tau}).$$

Vector primitives

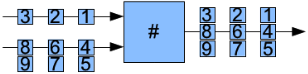
$$
\begin{aligned}
T(\texttt{vectorize}) \quad = \quad & \Lambda f : Rate.\tau : Type.n : \mathbb{N}^*. \\
& (\tau^f, Int[n,n]^0) \rightarrow (vector_n(\tau)^{f/n})
\end{aligned}
$$



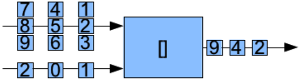
$$
T(\texttt{serialize}) = \Lambda f : Rate.\tau : Type.n : \mathbb{N}^*.(vector_n(\tau)^f) \rightarrow (\tau^{n.f})
$$



$$T(\#) \quad = \quad \Lambda f : Rate.\tau : Type.\tau' : Type.n : \mathbb{N}^*.n' : \mathbb{N}^*.$$
$$(vector_n(\tau)^f, vector_{n'}(\tau')^f) \rightarrow (vector_{n+n'}(\tau \sqcup \tau')^f)$$



$$T([\,]) = \Lambda f : Rate.\tau : Type.n : \mathbb{N}^*.(vector_n(\tau)^f, Int[0, n-1]^f) \rightarrow (\tau^f)$$



Semantic rules

$$(\mathrm{i})\ \frac{T(\mathtt{I}) = \Lambda l.z \to z' \qquad \forall (x,S) \in l, \quad l'[l^{-1}(x,S)] \in S}{T \vdash \mathtt{I} : (z \to z')[l'/l]}$$

$$(\subset)\ \frac{T \vdash E : z \to z' \qquad z_1 \subset z \qquad z' \subset z_1'}{T \vdash E : z_1 \to z_1'}$$

$$(,)\ \frac{T \vdash E_1 : z_1 \to z_1' \qquad T \vdash E_2 : z_2 \to z_2'}{T \vdash E_1, E_2 : z_1 || z_2 \to z_1' || z_2'}$$

$$(:)\ \frac{T \vdash E_1 : z_1 \to z_1' \qquad T \vdash E_2 : z_1' \to z_2'}{T \vdash E_1 : E_2 : z_1 \to z_2'}$$

Polymorphism and overloading

- All primitives are polymorphic due to abstractions in type schemes
- :> adds vector signals pointwise
- Overloading of arithmetic operators

Faust process

Definition

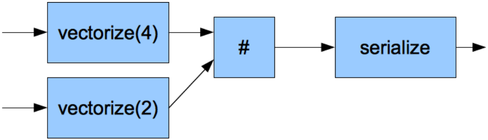
A Faust process is a well-typed expression such that all its I/O signals are scalar and of non-zero frequency.

The non-zero frequency condition ensures that all vector dimensions are known at compile time.

Goal

Static type inference of annotation free code.

```
vectorize(4),vectorize(2) : # : serialize
```



$$(\tau^{4f}, \tau'^{2f}) \rightarrow ((\tau \sqcup \tau')^{6f})$$

Type representation, environment

Isomorphic representation of types :

$$t : \text{Type} \to \text{Range} \times \text{Dimension}$$

$$\begin{array}{rcll} d \in \text{Dimension} & = & \mathit{Scalar} & (\text{for } \mathrm{b}[x, y]) \\ & | & n :: d' & (\text{for } \mathit{vector}_n(\tau)) \end{array}$$

For instance, $t(\mathit{vector}_3(\mathit{vector}_2(\mathit{Int}[0, 1]))) = (\mathit{Int}[0, 1], [3, 2])$.

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$$\begin{aligned} \_ \quad &\mapsto \quad (r, d, f) \rightarrow (r, d, f), \emptyset \\ 0 \quad &\mapsto \quad () \rightarrow (\mathit{Int}[0,0], \mathit{Scalar}, 0), \emptyset \end{aligned}$$

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Algorithm: constraint generation

$$
\begin{array}{rcl}
\texttt{type}(E, L_0) & = & \textit{match } E \textit{ with} \\
\texttt{I} & \mapsto & \texttt{New}\ (\textit{Env}\ (\texttt{I}), L_0) \\
E_1, E_2 & \mapsto & (I_1 || I_2 \rightarrow O_1 || O_2), \mathcal{C}_1 \cup \mathcal{C}_2, L_2 \\
E_1 : E_2 & \mapsto & (I_1 \rightarrow O_2), \mathcal{C}_1 \cup \mathcal{C}_2 \cup \textit{subbeam}(O_1, I_2), L_2 \\
\ldots & &
\end{array}
$$

where $(I_i \rightarrow O_i, \mathcal{C}_i, L_i) = \texttt{type}(E_i, L_{i-1})$,
$\texttt{New}$ creates a new instance of $\textit{Env}(\texttt{I})$ with fresh variables,
L_i is the set of used variables.

Constraints reduction

- Dimension equalities and frequency relations are reduced to numerical equalities and substitutions with inference systems:

$$\{\mathtt{d}_i = d\} \cup \mathcal{D}; \mathcal{N}; \mathcal{S} \Rightarrow \mathcal{D}[d/\mathtt{d}_i]; \mathcal{N}; \mathcal{S}[d/\mathtt{d}_i] \cup \{\mathtt{d}_i \mapsto d\}$$
$$\{n :: d \ = \ n' :: d'\} \cup \mathcal{D}; \mathcal{N}; \mathcal{S} \Rightarrow \{d = d'\} \cup \mathcal{D}; \mathcal{N} \cup \{n = n'\}; \mathcal{S}$$
$$\{Scalar \ = \ n :: d\} \cup \mathcal{D}; \mathcal{N}; \mathcal{S} \Rightarrow \text{fail}$$
$$\ldots$$

- Range relations can lead to
  - static computation of vector dimensions
  - static verification of arithmetic relations
  - dynamic clipping of signals

Correctness

Theorem (Soundness)

Let E be a Faust expression, and $(I \to O, \mathcal{C}, L) = \mathit{type}(E, \emptyset)$. Then, if $\mathcal{M}$ is a model defined on L such that $\mathcal{M} \models \mathcal{C}$, then $T \vdash E : t^{-1}(\mathcal{M}(I \to O))$.

Theorem (Completeness)

Let E be a Faust expression, such that $T \vdash E : z \to z'$. Then $\mathit{type}(E, \emptyset) = (I \to O, \mathcal{C}, L)$, and there exists a model $\mathcal{M}$ defined on L such that $\mathcal{M} \models \mathcal{C}$ and $t^{-1}(\mathcal{M}(I \to O)) \subset z \to z'$.

Conclusion

- Static inference of rate relations and vector dimensions.
  OCaml prototype.

  How to gain precision on data signal types?

- Expressive power of Faust

  Is the vector extension well suited for DSP algorithms?
  Study cases with IRCAM

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