# Quantum Chronodynamics Simulations and Efficient Evaluation of the Wilson-Dirac Operator

$$D\psi(x) = A\psi(x) - \frac{1}{2} \sum_{\mu=0}^{4} \{ [(I_4 - \gamma_\mu) \otimes U_{x,\mu}] \psi(x + \hat{\mu}) + [(I_4 + \gamma_\mu) \otimes U_{x-\hat{\mu},\mu}^{\dagger}] \psi(x - \hat{\mu}) \} \}$$

## Claude Tadonki

Mines ParisTech Centre de Recherche Informatique claude.tadonki@mines-paristech.fr





Maths & Systems SEMINAR Mines ParisTech, Bd Saint-Michel November 14, 2013, Paris (France)





**Q**uantum **C**hromo**D**ynamics (**QCD**) is the theory of strong interactions, whose ambition is to explain nuclei cohesion as well as neutron and proton structure, i.e. most of the visible matter in the Universe.

Creation of the universe (matter synthesis right after the BIG BANG) !!!



The main computational model for Quantum chromodynamics (QCD) investigations is the so-called Lattice Quantum ChromoDynamics (LQCD).

Based on the <u>LQCD model</u>, (large scale) <u>simulations</u> are implemented !!!



In the LQCD model, the space-time is represented by a finite discrete system with **cartesian coordinates**, while the interaction between subparticles is governed by strong force theory.



LQCD provides an <u>analytical formalism</u>, the reference for <u>numerical studies</u>.



With a **fixed lattice spacing**, **fine-grained** simulations are considered through **larger lattice** volume, thus increasing **memory** and **computation** complexities.

Solving the Wilson-Dirac system is the most critical computation kernel.





The Wilson-Dirac operator D on a site x (a spinor) can be defined

as  

$$D\psi(x) = A\psi(x) - \frac{1}{2} \sum_{\mu=0}^{4} \{ [(I_4 - \gamma_\mu) \otimes U_{x,\mu}] \psi(x + \hat{\mu}) + [(I_4 + \gamma_\mu) \otimes U_{x-\hat{\mu},\mu}^{\dagger}] \psi(x - \hat{\mu}) \} \}$$

where

ParisTech

- A is a  $12 \times 12$  complex matrix of the form  $\alpha I_{12} + \beta(\nu \otimes \gamma_5)$ , where  $\alpha, \beta$  are complex coefficients and  $\nu = 3 \times 3$  complex matrix
- x is a given point of the lattice, which is a finite subset of  $\mathbb{N}^4$
- $\psi$  (called *quark field* or *Wilson vector*) is a vector of 12-components complex vectors over the whole lattice
- $U_{x,\mu}$  is a 3 × 3 complex matrix (called *gluon field matrix* or *gauge matrix*) at  $(x,\mu)$ .

$$\begin{split} \gamma_0 &= \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \gamma_1 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} \\ \gamma_2 &= \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \gamma_3 = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix} \\ \gamma_5 &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \end{split}$$



8-neighbors stencil computation

The application of the Wilson-Dirac operator over all *spinors* of a *quark field* can be viewed as the product of a matrix-vector product, thus the *Wilson-Dirac matrix*.





#### Evaluation of the Wilson-Dirac Operator

MINES

ParisTech

Remark 1. Given u a 4-components complex vector,  $s \in \{-1, 1\}$ , and  $\mu \in \{0, 1, 2, 3\}$ , the 4-components complex vector v defined by

$$v = (I_4 - s\gamma_\mu)u,$$

can be more efficiently calculated using the following relations

	$\mu = 0$	$\mu = 1$	$\mu = 2$	$\mu = 3$	1
s = 1	$v_1 = u_1 + u_3$	$v_1 = u_1 + iu_4$	$v_1 = u_1 + u_4$	$v_1 = u_1 + iu_3$	I
	$v_2 = u_2 + u_4$	$v_2 = u_2 + iu_3$	$v_2 = u_2 - u_3$	$v_2 = u_2 - iu_4$	I
	$v_3 = v_1$	$v_3 = -iv_2$	$v_3 = -v_2$	$v_3 = -iv_1$	I
	$v_4 = v_2$	$v_4 = -iv_1$	$v_4 = v_1$	$v_4 = iv_2$	4
	$v_1 = u_1 - u_3$	$v_1 = u_1 - iu_4$	$v_1 = u_1 - u_4$	$v_1 = u_1 - iu_3$	
s = -1	$v_2 = u_2 - u_4$	$v_2 = u_2 - iu_3$	$v_2 = u_2 + u_3$	$v_2 = u_2 + iu_4$	I
	$v_3 = -v_1$	$v_3 = iv_2$	$v_3 = v_2$	$v_3 = iv_1$	I
	$v_4 = -v_2$	$v_4 = iv_1$	$v_4 = -v_1$	$v_4 = -iv_2$	1



Remark 2. Using the normal factors decomposition and its commutativity, we have

$$(I_4 - s\gamma_\mu) \otimes U = ((I_4 - s\gamma_\mu) \otimes I_3)(I_4 \otimes U)$$
$$= (I_4 \otimes U)((I_4 - s\gamma_\mu) \otimes I_3)$$

Thus, the computation of  $[(I_4 - s\gamma_\mu) \otimes U]\psi$  can be done as follows

$$[(I_4 - s\gamma_\mu) \otimes U]\psi = (I_4 \otimes U)\{((I_4 - s\gamma_\mu) \otimes I_3)\psi\}$$

> Using the above <u>factorizations</u> & simplifications leads to an optimal evaluation of Wilson-Dirac.

> Data locality is the main issue that will impact on memory and data communication costs.



8×N spinors	$8\times(24\times8)\times \texttt{N}$	$1536 \times N$ bytes
8×N U matrices	8  imes (18  imes 8)  imes N	$1152{ imes}\mathbb{N}$ bytes
$8 \times N$ integer indexes	8  imes (18)  imes N	$64{ imes}$ N bytes

N = LxLyLzLt

Fig. Data inventory for a single Dirac operator

- In order reduce the cost of irregular memory accesses related to SU(3) matrices, the so-called GAUGE-COPY strategy is considered: for each site of the whole lattice, the required 8 SU(3) matrices are stored contigously → better memory performance at the expense of redundancy!
- **Examples of GAUGE array sizes** (sizeof(U) = 3x3x2x8 = 144 bytes).

L	т	Sizeof(Us)
16	32	0.28 GB
32	64	4.5 GB
64	128	72 GB
128	256	1152 GB

Parallelism is considered also because of the <u>global memory requirement</u>, even if work inefficient!!!

**Efficient Wilson-Dirac** is vital for LQCD simulations as it is evaluated so many times.





Our goal is to solve the following linear system

 $D\psi(x) = \phi$ 

This system is an important step the synthesis of a statistical gauge configuration sample.

The solution of the system, called propagator, appears in the expression of the so-called *fermionic force*, used to update the momenta associated with the gauge fields along a trajectory in the Hybrid Monte Carlo (HMC) algorithm.

Many propagators are computed on a trajectory, they should fullfil the **reversibility criterion**.

- Large-scale simulations involve huge Dirac matrices with bad condition numbers for small pion masses.
- Because of the <u>reversibility</u> condition and the <u>quality expected for the propagators</u> in order to be useful for physic, a <u>high numerical precision</u> is required to solve the Dirac system.
- Because of the implicit form of the Dirac matrix and the required precision, iterative methods are considered. We should prepare for a huge number of iteration or divergence.

Large-scale inversions need a combination of state-of-the-art HPC and matrix computation !!!

LQCD research is actiove through collaborations (projects) and dedicated supercomputers.





- Large volume of data ( disk / memory / network )
- Significant number of solvers iterations due to numerical intractability
- Redundant memory accesses coming from interleaving data dependencies
- Use of double precision because of accuracy need (hardware penalty)
- Misaligned data (inherent to specific data structures)
  - Exacerbates cache misses (depending on cache size)
  - Becomes a serious problem when consider accelarators
  - Leads to « false sharing » with Shared-Memory paradigm (Posix, OpenMP)
  - Padding is one solution but would dramatically increase memory requirement
- Memory/Computation compromise in data organization (e.g. gauge replication)





- High-precision Lattice Quantum ChromoDynamics simulations.
- The ANR project **PetaQCD** was targeting **256×128<sup>3</sup>** lattices.
- One evaluation of the *Dirac operator* on a  $256 \times 128^3$  lattice involves 256 × 128<sup>3</sup> × 1500 ≈ 10<sup>12</sup> (stencil) floating-point operations



#### Important Facts about Supercomputers

MINES

ParisTech

## Claude TADONKI

PFlops





TITAN CRAY-XK7 the (2012) world fastest supercomputer

- 299 008 CPU cores (16-cores AMD Opteron 6274)
- 18 688 NVIDIA Tesla K20 GPUs

TFlops

- Peak: 27.11 PFlop/s.
- Sustained: 17.59 PFlop/s (Linpack)



#### Supercomputers for LQCD (and stencil applications)





#### LQCD Simulations and Efficient Wilson-Dirac Operator Maths & Systems SEMINAR, Mines ParisTech, Bd Saint-Michel

November 14, 2013, Paris (France)





#### IBM System Technology Group Examples of Applications Running on Blue Gene

Developed on L, P; many ported to Q

Application	Owner	Application	Owner	Application	Owner
CFD Alya System	Barcelona SC	DFT iGryd	Jülich	BIM: SPEC2006, SPEC openmp	SPEC
CFD (Flame) AVBP	CERFACS Consortium	DFT KKRnano	Jülich	BM: NAS Parallel Benchmarks	NASA
CFD dns3D	Argonne National Lab	DFT ls3df	Argonne National Lab	BM: RZG (AIMS,Gadget,GENE, GROMACS,NEMORB,Octopus, Vertex)	RZG
CFD OpenFOAM	SGI	DFT PARATEC	NERSC / LBL	Coulomb Solver - PEPC	Jülich
CFD NEK5000, NEKTAR	Argonne, Brown U	DFT CPMD	IBM/Max Planck	MPI PALLAS	UCB
CFD OVERFLOW	NASA, Boeing	DFT QBOX	<b>LINL</b>	Mesh AMR	CCSE, LBL
CFD Saturne	EDF	DFT VASP	U Vienna & Duisburg	PETSC	Argonne National Lab
CFD LB M	Erlanger-Nuremberg	Q Chem GAMESS	Ames Lab/Iowa State	MpiBlast-pio Biology	VaTech / ANL
MD Amber	UCSF	Nuclear Physics GFMC	Argonne National Lab	RTM-Seismic Imaging	ENI
MD Dalton	Univ Oslo/Argonne	Neutronics SWEEP3D	LANL	Supernova la FLASH	Argonne National Lab
	LINL	QCD CPS	Columbia U/IBM	Ocean HYCOM	NOPP / Consortium
MD LAMMPS	Sandia National Labs	QCD MLC	Indiana University	Ocean POP	LANL/ANL/NCAR
MD MP2C	Jülich	Plasma GTC	PPPL	Weather/Climate CAM	NCAR
MDNAMD	UIUC/NCSA	Plasma GYRO (Tokamak)	General Atomics	Weather/Climate Held-Suarez Test	GFDL
MD Rosetta	U Washington	KAUST Stencil Code Gen	KAUST	Climate HOMME	NCAR
DFT GPAW	Argonne National Lab	BM:sppm,raptor,AMG,IRS,sphot	Livermore	Weather/Climate WRF, CM1	NCAR, NCSA

#### Accelerating Discovery and Innovation in:









Climate & Environment



Life Sciences

Whole Organ Simulation

Silicon Design

- Next Gen Nuclear
- **High Efficiency Engines**
- Oil Exploration









An accelerator based engine (IBM / SONY / TOSHIBA)

25.6 GB/s memory bandwidth

Fast SPE -> SPE communication

<u>1 PPE</u>: 2-way SMT PowerPC core, 3.2 Ghz, Up to 32 GB DDR2 <u>8 SPEs</u>: 3.2 GHz VMX vector unit, 128 SIMD, 256 Kb SRAM 25.6 x p GFLOPS SP / 12.8 x p GFLOPS DP, p = 8, 16, ...

Asynchronous PPE->SPE / SPE -> SPE communication

## Why the CELL Processor ?

Highest computing power in a single « computing node »

Fast memory access

Asynchronysm between data transfers and computation

## Issues with the CELL Processor ?

- Data alignment (both for calculations and transfers)
- Heavy use of list DMA
- Small size of the Local Store (SPU local memory)
- Ressources sharing with Dual Cell Based Blade
- Integration into an existing standard framework

LQCD Simulations and Efficient Wilson-Dirac Operator

Maths & Systems SEMINAR, Mines ParisTech, Bd Saint-Michel November 14, 2013, Paris (France)





#### What we have done

Implementation of each critical kernel on the CELL processor
 SIMD version of basic operators
 Appropriate DMA mechanism (efficient list DMA and double buffering)

Merging of consecutive operations into a unique operator (latency & memory reuse)

Aggregation of all these implementations into a single and standalone library
 A single SPU thread holds the whole set of routines
 SPU thread remains « permanently » active during a working session

- Effective integration into the tmLQCD package
  - Data re-alignment

Routine calls replacement (invoke CELL versions in place of native ones) This should be the way to commit this back to tmLQCD (external library and « IsCELL » switch)

Successful tests (QS20 and QS22)





#### Global organization of the computation

Task partitioning, distribution, and synchorization are done by the PPU



(DMA get + SIMD Computation + DMA put)



GENERIC LQCD COMPUTATION MECHANISM ON THE CELL

The SPE, always active, switches to the appropriate operation on each request







#### Performance results



We consider a 32x16<sup>3</sup> lattice and CELL-accerated version of tmLQCD using our code (double precision)

QS20				
#SPE	Time(s)	Speedup	GFlops	
1	0.109	1.00	0.95	
2	0.054	2.00	1.92	
3	0.036	3.00	2.89	
4	0.027	3.99	3.85	
5	0.022	4.98	4.73	
6	0.018	5.96	5.78	
7	0.015	6.93	6.94	
8	0.013	7.88	8.01	

	QS22				
#SPE	Time(s)	Speedup	GFlops		
1	0.0374	1.00	2.76		
2	0.0195	1.91	5.31		
3	0.0134	2.79	7.76		
4	0.0105	3.56	9.90		
5	0.0090	4.81	13.27		
6	0.0081	5.75	15.87		
7	0.0076	6.84	18.88		
8	0.0075	0075 7.82 2			

INTEL i7 quadcore 2.83 Ghz				
Witho	ut SSE	With SSE		
1 core 4 cores		1 core	4 cores	
0.0820 0.0370		0.040	0.0280	

More than 80% efficiency on a QS20 !!!

	INTEL i7	CELL (8 SPEs)	
	SSE + 4c	QS20	QS22
GCR (57 iters)	11.05 s	3.78 s	2.04 s
CG (685 iters)	89.54 s	42.25 s	22.78 s

ICS-HEART 2010 Epochal Tsukuba, Tsukuba, Japan, June 1, 2010





#### Claude TADONKI

## Preliminary remark about Wilson-Dirac inversion

- We are involved with (very) large-scale linear systems with an implicit principal matrix.
- Because of the implicit form of the Dirac matrix, we only use iterative methods.
- Sor some parameters (e.g. light masses), the Dirac matrix has a poor condition number.
- One way to tackle ill-conditioned cases is to use the so-called deflation method.
- The deflation method attempts to « isolate » the low modes space.
  - The **deflation method** is a *two stages* iterative method.

In order to be efficient, both from the qualitative and the quantitative point of view, the deflation method needs a **good calibration** and lot of **programming efforts**.









## TECHNICAL OBSERVATIONS

- The need to approximate the (nearly) kernel subspace.
- Our approximation of the low modes space concerns both the **dimension** and the **basis**.
- Solution (over/under)dimensioned approximation will lead to degrade the global performance.
- The quality of the «low» eigenvectors approximations is also concerned (although the common belief that this is not important).
- The second stage subsystem is also solved iteratively.
- The number of iterations of global system and that of the inner system are correlated.

With large gauge configurations and low masses, we have a huge number of iterations and each iteration is costly, thus we are facing a large-scale computation problem.







#### APPROXIMATION OF THE LOW MODES SUBSPACE (also called DEFLATION SUBSPACE)



The dimension of the deflation subspace is given as a user parameter  $(N_s)$ .



The computation of the deflation subspace basis (i.e.  $N_s$  approximated eigenvectors) is severely costly.

M We choose to compute each of the  $N_s$  vectors as precise as possible (heavy!!!).

It is expected that the more the deflation subspace is accurate the shorter will be the path to the solution of the global system. This is true in theory, but the reality is versatile.



One argument commonly admitted to pay the price for a good deflation subspace is that this will serve for several inversions.

Since our system is large, we need a way to extend the deflation basis at a reasonable cost.

We choose to extend the deflation subspace by means of canonical block projections.





## BLOCK DECOMPOSITION AND THE LITTLE DIRAC

- The whole lattice divided into blocks  $\Theta(\vec{b})$  on a four dimensional grid.
- Every block has a grid coordinate  $\vec{b}$  and we have a total number of  $N_b$  blocks.
- Assume we have found  $N_s$  approximate (global) eigenvectors  $\psi_l, l = 1, ..., N_s$ .
- We decompose them (by restrictions) onto to the blocks as follows:

$$\phi_l^{\vec{b}}(x) = \begin{cases} \psi_l(x) & \text{if } x \in \Theta(\vec{b}) ,\\ 0 & \text{otherwise} . \end{cases}$$

• We so construct  $N_b \cdot N_s$  vectors, which are orthonormalised afterwards.

The *little Dirac operator* A is computed as follows:  $A_{(\vec{a},k)(\vec{b},l)} = \langle \phi_k^{\vec{a}} | D \phi_l^{\vec{b}} \rangle$ 





## OUR WORK ON THE BLOCK DECOMPOSITION

We have implemented a generic block decomposition (multidimensional and with no restriction on the number of blocks per axis) within the tmLQCD package.

For this purpose, we adapted the following modules

- The input parameters list and associated I/O routines.
- The initialization routines.
- The extension of the initial pseudo-eigenvectors using block decomposition.
- The preparation of the little Dirac components.
- The checking routines (to make sure we have good projectors).
- The computation of the little Dirac and its product by a vector.
- MPI communication according to the decomposition.
- Mechanism to store (and later read) the generated intial pseudo-eigenvectors.





#### EXPERIMENTAL RESULTS

				E	Experimental	results	of the deflation using tmLQCD
			Exp	eriments perfor R	med with a 64×3: emark: The numb	2 <sup>3</sup> lattice DflFieldIte S er of inne	C. Tadonki (critical kappa = 0.154073) using 128 cores on Jade (cines) er = 80 DflPolyIter = 20 Get the code Makefile ample input file er iterations need to be carefully examined
N٥	kappa	2KappaMu	Method	Block partition	N° of iterations	Time(s)	
1	0.1537650	0	cg	-	2424	267	
2	0.1537650	0	dflgcr	(1, 1, 2, 2)	46	80	
3	0.154073	0.002003	cg	-	783	87	
4	0.154073	0.002003	dflgcr	(8, 2, 2, 4)	137	286	
5	0.154073	0.002003	dflgcr	(4, 2, 2, 2)	169	299	
6	0.154073	0.002003	dflgcr	(2, 2, 2, 4)	198	350	
7	0.154073	0.002003	dflgcr	(2, 2, 4, 2)	189	332	
8	0.154073	0.002003	dflgcr	(2, 4, 4, 2)	175	323	
9	0.154073	0.002003	dflgcr	(2, 2, 2, 2)	214	370	
10	0.154073	0.002003	dflgcr	(2, 2, 4, 4)	175	323	
11	0.154073	0.002003	dflgcr	(2, 4, 2, 4)	176	328	
12	0.154073	0.002003	dflgcr	(4, 2, 2, 4)	153	282	
13	0.154073	0.002003	dflgcr	(2, 4, 2, 2)	196	347	
14	0.154073	0.002003	dflgcr	(8, 4, 4, 4)	112	481	
15	0.154073	0.002003	dflgcr	(8, 2, 4, 2)	137	280	
16	0.154073	0.002003	dflgcr	(4, 2, 4, 4)	144	295	
17	0.154073	0.002003	dflgcr	(8, 4, 2, 4)	124	328	
18	0.154073	0.002003	dflgcr	(2, 4, 4, 4)	166	341	
19	0.154073	0.002003	dflgcr	(4, 4, 4, 4)	135	351	
20	0.154073	0.002003	dflgcr	(8, 2, 2, 2)	148	272	
21	0.154073	0.002003	dflgcr	(4, 4, 2, 4)	143	293	
22	0.154073	0.002003	dflgcr	(8, 2, 4, 4)	124	326	





#### Why an Automatic Code Generation System



As the formulae could be frequently changed or adapted, a push-button system to get the corresponding code is certainly comfortable.



One application of the Dirac operator involves more than thousand floating point instructions, thus it is hard to plan low level optimization by hand.

There are <u>different variants</u> of the Dirac operator and several way of expression the calculation depending on the <u>data layout</u>.

 $\bigcirc$ 

There are <u>different target architectures</u> which can be considered. Thus, generating the code for each of them manually can be <u>tedious</u> and <u>error-prone</u>.

One way to seek optimal implementation is to filter from an exhaustive search over all possible (or reasonable).



D. Barthou, G. Grosdidier, M. Kruse, O. Pene and C. Tadonki,

« QIRAL: A High Level Language for Lattice QCD Code Generation »,

Programming Language Approaches to Concurrency and Communication-cEntric Software (<u>PLACES'12</u>) in conjunction with the European joint Conference on Theory & Practice of Software (<u>ETAPS</u>), Tallinn, March 24-April 1, 2012.



LQCD Simulations and Efficient Wilson-Dirac Operator Maths & Systems SEMINAR, Mines ParisTech, Bd Saint-Michel

November 14, 2013, Paris (France)



## The needs from the pseudo-code to an valid C code

- the variables need to be explicitly declared (QIRAL declares part of the main ones)
- some simplification still need to be done  $(id(C) \times id(S) = id(C \times S)$  and  $id(n)^*u = u$ )
- special statements need to be appropriately expanded (sum(d in {dx,dy,dz,dt}))
- Iibraries calls are required for macroscopic operations (Ap[L].Ap[L]); (Ap[L].r[L])
- specific routines should be called for special operations like (id(S) + gamma(d))\*u)
- 4D indexation should be correctly handled/matched with its 1D correspondence
- some profiling and monitoring instructions should be inserted for user convenience
- I/O routines are required for user parameters and data files
- Output the second se

The last point is particularly important since it will determine the routines to be called (in standard C) or the method execute (C++ / ad-hoc polymorphism).





## The C code from the pseudo-code

## The module is based on *Lex* and *Yacc*

- Its generated a valid C code
- The output is associated to an external library for special routines
- A C++ output is planned, will be associated with QDP++ or QUDA through a specific interface
- A web based interface is available

f(spinor_source!=NULL) memcpy(bb,spin	or_source,VOLUME*sizeof(spinor));
(U0!=NULL) memcpy(U,U0,8*VOLUME*s	izeof(su3));
epsilon = qiral_epsilon;	
appa = qiral_kappa;	
nu = qiral_mu;	
<pre>tx = index_1D(1, 0, 0, 0); /* one step in</pre>	the direction x */
<pre>dy = index_1D(0, 1, 0, 0); /* one step in</pre>	the direction y */
<pre>index_1D(0, 0, 1, 0); /* one step in</pre>	the direction z */
<pre>index_1D(0, 0, 0, 1); /* one step in 1</pre>	the direction t */
"End of internal initializations "/	
or(t = 0 ; t < LT ; t ++ )	
{for(z = 0 ; z < LZ ; z ++ )	
{for(y = 0 ; y < LY ; y ++ )	
{for(x = 0 ; x < LX ; x ++ )	
{	
<pre>s = index_1D(x, y, z, t); /* linearizat</pre>	ion */
r[s] = bb[s];	
p[s] = r[s];	
<pre>x1[s] = spn_zero();</pre>	
<pre>}}}</pre>	
nr = square_norm(r, L, 0);	
or(t = 0; t < LT; t ++)	
{for(z = 0; z < LZ; z ++)	
$\{for(y = 0; y < LY; y ++)\}$	
{for(x = 0 ; x < LX ; x ++ )	
t - index 10% and 20 (5 lineariest	
s = index_1D(x, y, z, t); /* inearizat	Jon -/
ID2 = mat_mul_spn(id_tensor((gm_	add_gm(gamma_mat(5), diag((I_dbi(mu ~ kappa ~ 2))))), p[s]);
ID1 = (_sph(ID2);	encer(III) an add an(annan mat/E) an mul an(annan ma
IDS = i_son(dbl_mul_son(kappa_ID)	ensor(o[u_up(s, 1)], gm_adu_gm(gamma_mac(s), gm_mu_gm(gamma_ma s));
ID32 = con add con(mat mul con)	(teneor/III) dole 1)] am eub amlasmos mot(5) am mul amlasmos m
ID22 = i spo(dbl_mul_spo(kappa_II	censor(o[u_un(s, 1)], gin_sub_gin(gamma_mac(s), gin_mui_gin(gamma_ma 022)):
IDS = son add son(ID22 ID5)	/22]],
$\Delta n[s] = snn add snn(ID1, ID5);$	
3333	
while(nr > epsilon)	
	RUN download s

PetaQCD





#### **Experimental results** Experimental results of the deflation using tmLQCD C. Tadonki Experiments performed with a $64 \times 32^3$ lattice (critical kappa = 0.154073) using 128 cores on Jade (cines) DflFieldIter = 80 DflPolyIter = 20 Get the code Makefile Sample input file Remark: The number of inner iterations need to be carefully examined N° kappa 2KappaMu Method Block partition N° of iterations Time(s) 1 0.1537650 0 2424 267 cg 80 2 0.1537650 0 dflacr (1, 1, 2, 2)46 87 3 0.154073 0.002003 783 ca 0.154073 0.002003 dflgcr (8, 2, 2, 4)137 286 299 5 0.154073 0.002003 dflgcr (4, 2, 2, 2)169 6 0.154073 0.002003 dflacr (2, 2, 2, 4)198 350 0.154073 0.002003 dflgcr (2, 2, 4, 2)189 332 7 8 0.154073 0.002003 dflgcr (2, 4, 4, 2)175 323 9 0.154073 0.002003 dflgcr (2, 2, 2, 2)214 370 323 10 0.154073 0.002003 dflgcr (2, 2, 4, 4)175 11 0.154073 0.002003 dflgcr (2, 4, 2, 4)176 328 12 0.154073 0.002003 dflgcr (4, 2, 2, 4)153 282 347 13 0.154073 0.002003 dflgcr (2, 4, 2, 2)196 14 0.154073 0.002003 dflgcr (8, 4, 4, 4)112 481 15 0.154073 0.002003 dflgcr (8, 2, 4, 2)137 280 16 0.154073 0.002003 dflgcr (4, 2, 4, 4)144 295 17 0.154073 0.002003 dflgcr (8, 4, 2, 4)124 328 18 0.154073 0.002003 dflgcr (2, 4, 4, 4)166 341 19 0.154073 0.002003 dflgcr (4, 4, 4, 4)135 351 20 0.154073 0.002003 dflgcr (8, 2, 2, 2) 148 272 21 0.154073 0.002003 dflgcr (4, 4, 2, 4)143 293 22 0.154073 0.002003 dflgcr (8, 2, 4, 4)124 326





#### PetaQCD (LQCD)

## Claude TADONKI



#### END & QUESTIONS





